$\boxed{2.10.1}$

$$T_A e_1 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} a \\ c \end{pmatrix},$$

同様に. $T_A e_2 = \begin{pmatrix} b \\ d \end{pmatrix}$, $T_B e_1 = \begin{pmatrix} \alpha \\ \gamma \end{pmatrix}$, $T_B e_2 = \begin{pmatrix} \beta \\ \delta \end{pmatrix}$

よって.

$$T_A \otimes T_B (e_1 \otimes e_1) = \begin{pmatrix} a \\ c \end{pmatrix} \otimes \begin{pmatrix} \alpha \\ \gamma \end{pmatrix}$$

$$= \left\{\begin{pmatrix} a \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ c \end{pmatrix}\right\} \otimes \left\{\begin{pmatrix} \alpha \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \gamma \end{pmatrix}\right\}$$

$$= \begin{pmatrix} a \\ 0 \end{pmatrix} \otimes \begin{pmatrix} \alpha \\ 0 \end{pmatrix} + \begin{pmatrix} a \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ \gamma \end{pmatrix} + \begin{pmatrix} 0 \\ c \end{pmatrix} \otimes \begin{pmatrix} \alpha \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ c \end{pmatrix} \otimes \begin{pmatrix} 0 \\ \gamma \end{pmatrix}$$

$$= a\alpha\, e_1 \otimes e_1 + a\gamma (e_1 \otimes e_2) + c\alpha (e_2 \otimes e_1) + c\gamma (e_2 \otimes e_2)$$

同様に $T_A \otimes T_B (e_1 \otimes e_2) = \begin{pmatrix} a \\ c \end{pmatrix} \otimes \begin{pmatrix} \beta \\ \delta \end{pmatrix}$

$$= a\beta\, e_1 \otimes e_1 + a\delta\, e_1 \times e_2 + c\beta\, e_2 \otimes e_1 + c\delta\, e_2 \times e_2$$

$$T_A \otimes T_B (e_2 \otimes e_1) = \begin{pmatrix} b \\ d \end{pmatrix} \otimes \begin{pmatrix} \alpha \\ \gamma \end{pmatrix}$$

$$= b\alpha\, e_1 \otimes e_1 + b\gamma\, e_1 \otimes e_2 + d\alpha\, e_2 \otimes e_1 + d\gamma\, e_2 \otimes e_2$$

$$T_A \otimes T_B (e_2 \otimes e_2) = \begin{pmatrix} b \\ d \end{pmatrix} \otimes \begin{pmatrix} \beta \\ \delta \end{pmatrix}$$

$$= b\beta\, e_1 \otimes e_1 + b\delta\, e_1 \times e_2 + d\beta\, e_2 \otimes e_1 + d\delta\, e_2 \otimes e_2$$

よって求める表現行列は

$$\begin{pmatrix} a\alpha & a\beta & b\alpha & b\beta \\ a\gamma & a\delta & b\gamma & b\delta \\ c\alpha & c\beta & d\alpha & d\beta \\ c\gamma & c\delta & d\gamma & d\delta \end{pmatrix}$$