$\boxed{4.2.2}$ x_1, x_2, x_3 をそれぞれ x, y, z に書き換えるとする.

(1)

$$
\delta(1,3)(x) = \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^3 \end{vmatrix}
$$

$$
= \begin{vmatrix} 1 & 0 & 0 \\ x & y-x & z-x \\ x^3 & y^3-x^3 & z^3-x^3 \end{vmatrix}
$$

$$
= \begin{vmatrix} y-x & z-x \\ y^3-x^3 & z^3-x^3 \end{vmatrix}
$$

$$
= (y-x)(z-x) \begin{vmatrix} 1 & 1 \\ y^2+yx+x^2 & z^2+zx+x^2 \end{vmatrix}
$$

$$
= (y-x)(z-x) \begin{vmatrix} 1 & 0 \\ y^2+yx+x^2 & z^2+zx-yx-y^2 \end{vmatrix}
$$

$$
= (y-x)(z-x)(z^2+zx-yx-y^2)
$$

$$
= (y-x)(z-x)\{(z-y)(z+y)+(z-y)x\}
$$

$$
= (y-x)(z-x)(z-y)(z+y+x)
$$

$$
= -(x-y)(x-z)(y-z)(x+y+z)
$$

$$
= -\delta(x,y,z)\, S_1
$$

(2)

$$
\delta(2,3)(x) = \begin{vmatrix} 1 & 0 & 0 \\ x^2 & y^2-x^2 & z^2-x^2 \\ x^3 & y^3-x^3 & z^3-x^3 \end{vmatrix}
$$

z^2+zx $\cancel{x^2}$

y^2+yx

$z^2-y^2+zx-yx$

$$
= (y-x)(z-x) \begin{vmatrix} y+x & z+x \\ y^2+yx+x^2 & z^2+zx+x^2 \end{vmatrix}
$$

$$
= (y-x)(z-x) \begin{vmatrix} y-z & z+x \\ (y-z)(x+y+z) & z^2+zx+x^2 \end{vmatrix}
$$

$$
= (y-x)(z-x)(y-z) \begin{vmatrix} 1 & z+x \\ x+y+z & z^2+zx+x^2 \end{vmatrix}
$$

$$
= \delta(x,y,z)\,(z^2+zx+x^2-zx-zy-z^2-x^2-xy-xz)
$$

$$
= \delta(x,y,z)\,(-zy-xy-xz)
$$

$$
= -\delta(x,y,z)\, S_2
$$