$\boxed{4.12.2}$ (1) $\mathrm{ch}\,\mathbb{Q} = 0$ より $[\mathbb{Q}(\alpha):\mathbb{Q}]_i = 1$,

よって $[\mathbb{Q}(\alpha):\mathbb{Q}]_s = |\mathrm{Hom}^{al}_{\mathbb{Q}}(\mathbb{Q}(\alpha), \overline{\mathbb{Q}})| = 3$

$x^3 - x + 1$ の α の共役を $\alpha_1, \alpha_2, \alpha_3$ とおくと. $(\alpha_1 = \alpha)$

$$\begin{aligned}
\mathrm{Tr}_{K/\mathbb{Q}}(\alpha+1) &= \alpha_1 + 1 + \alpha_2 + 1 + \alpha_3 + 3 \\
&= (\alpha_1 + \alpha_2 + \alpha_3) + 3 \\
&= 3.
\end{aligned}$$

$$\begin{aligned}
N_{K/\mathbb{Q}}(\alpha+1) &= (\alpha_1+1)(\alpha_2+1)(\alpha_3+1) \\
&= -f(-1) \\
&= -(-1+1+1) \\
&= -1
\end{aligned}$$

$f(x) = (x-\alpha_1)(x-\alpha_2)(x-\alpha_3)$ より

$f(-1) = -(1+\alpha_1)(1+\alpha_2)(1+\alpha_3)$

であるから.

(2).
$$\begin{aligned}
\mathrm{Tr}_{K/\mathbb{Q}}(\alpha^2-\alpha) &= \alpha_1^2 - \alpha_1 + \alpha_2^2 - \alpha_2 + \alpha_3^2 - \alpha_3 \\
&= \alpha_1^2 + \alpha_2^2 + \alpha_3^2 - (\alpha_1 + \alpha_2 + \alpha_3) \\
&= (\alpha_1 + \alpha_2 + \alpha_3)^2 - 2(\alpha_1\alpha_2 + \alpha_2\alpha_3 + \alpha_3\alpha_1) \\
&= -2 \times (-1) \\
&= 2
\end{aligned}$$

$$\begin{aligned}
N_{K/\mathbb{Q}}(\alpha^2-\alpha) &= (\alpha_1^2-\alpha_1)(\alpha_2^2-\alpha_2)(\alpha_3^2-\alpha_3) \\
&= \alpha_1\alpha_2\alpha_3(\alpha_1-1)(\alpha_2-1)(\alpha_3-1) \\
&= -1 \cdot \{-f(1)\} \\
&= -1 \times (-1) \\
&= 1
\end{aligned}$$