

4.2.1

λ, μ, ν は任意. x, y, z は変数

$f(x, y, z)$ は x, y, z の関数

$$\begin{aligned} (1) \quad S_1 S_2 &= (x+y+z)(xy+yz+zx) \\ &= x^2y + x^2z + xy^2 + y^2z + x^2y + x^2z + yz^2 + z^2x \\ &= x^2y + x^2z + xy^2 + y^2z + yz^2 + z^2x + 3xyz \quad (1') \end{aligned}$$

$$\begin{aligned} f(x, y, z) - S_1 S_2 &= -3xyz \\ &= -3S_3 \end{aligned}$$

よって $f(x, y, z) = S_1 S_2 - 3S_3$

$$\begin{aligned} (2) \quad S_2^2 &= (xy+yz+zx)^2 \\ &= x^2y^2 + y^2z^2 + z^2x^2 + 2xy^2z + 2x^2yz + 2xy^2z \quad (1'') \\ f(x, y, z) - S_2^2 &= -2S_1 S_3 \end{aligned}$$

よって $f(x, y, z) = S_2^2 - 2S_1 S_3$

$$\begin{aligned} (3) \quad S_1^2 S_2 &= (x+y+z)^2 (xy+yz+zx) \\ &= (x^2+y^2+z^2+2xy+2yz+2zx)(xy+yz+zx) \\ &= \underline{x^3y} + \underline{x^2y^2} + x^2yz^2 + \underline{2x^2y^2} + 2xy^2z + 2x^2yz \\ &\quad + x^2yz + \underline{y^3z} + \underline{yz^3} + 2xy^2z + \underline{2y^2z^2} + 2xy^2z^2 \\ &\quad + \underline{x^3z} + x^2yz + \underline{xz^3} + 2x^2yz + 2xy^2z^2 + \underline{2x^2z^2} \end{aligned}$$

$$\begin{aligned} f(x, y, z) - S_1^2 S_2 &= -2(x^2y^2 + y^2z^2 + x^2z^2) - xyz(z + 2y + 2x + x + 2y + 2z + z + 2x + 2z) \\ &= -2S_2^2 + 4S_1 S_3 - S_3 \cdot 5S_1 \end{aligned}$$

$$\begin{aligned} (21'') \quad &= -2S_2^2 - S_1 S_3 \\ &= -2S_2^2 - S_1 S_3 \end{aligned}$$

よって $f(x, y, z) = S_1^2 S_2 - 2S_2^2 - S_1 S_3$